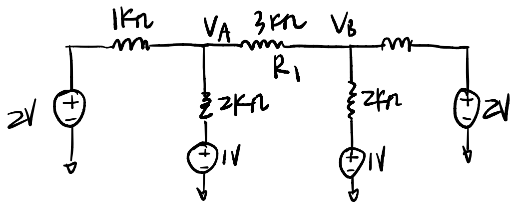
1. Solution:



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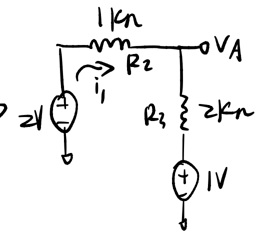
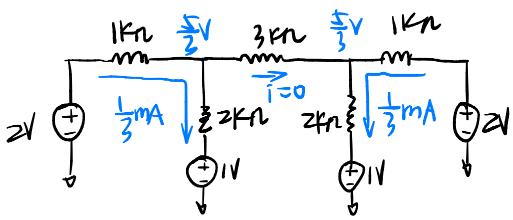


Because of symmetry, $V_A = V_B$, and there is no current flowing through R_1.

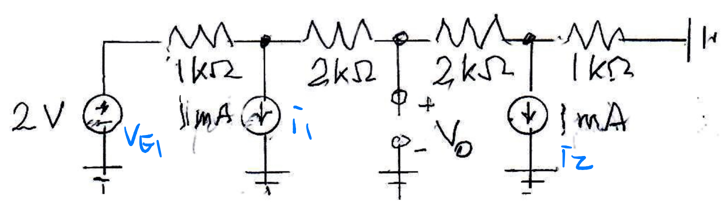
$$i_1 = \frac{2V - 1V}{1k\Omega + 2k\Omega} = \frac{1}{3} mA$$

$$V_A = 2V - 1k\Omega \times \frac{1}{3} mA = \frac{5}{3} V.$$

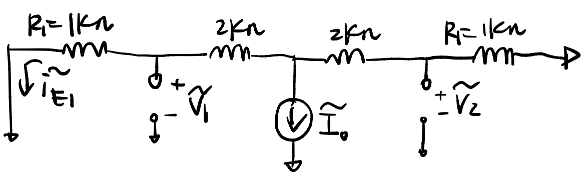


2. Solution:

circuit N:



circuit $\tilde{N}$:



$$\Rightarrow \begin{cases} \tilde{i}_{E1} = -\frac{1}{2}\tilde{I}_o \\ \tilde{V}_1 = -\frac{1}{2}\tilde{I}_o R_1 \\ \tilde{V}_2 = -\frac{1}{2}\tilde{I}_o R_1 \end{cases}$$

Using inter-reciprocity, $V_o \cdot \tilde{I}_o + V_{E1} \cdot \tilde{i}_{E1} = i_1 \cdot \tilde{V}_1 + i_2 \cdot \tilde{V}_2$

$$\Rightarrow V_o = -V_{E1} \cdot \frac{\tilde{i}_{E1}}{\tilde{I}_o} + i_1 \cdot \frac{\tilde{V}_1}{\tilde{I}_o} + i_2 \cdot \frac{\tilde{V}_2}{\tilde{I}_o}$$

$$= \frac{1}{2} V_{E1} - \frac{1}{2} R_1 i_1 - \frac{1}{2} R_1 i_2$$

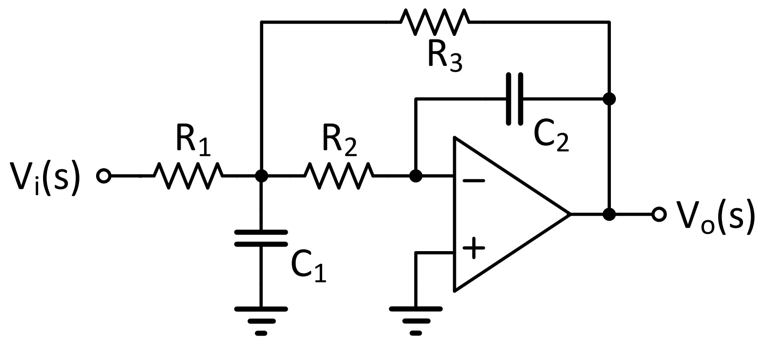
$$= \frac{1}{2} \times 2V - \frac{1}{2} \times 1k\Omega \cdot 1mA - \frac{1}{2} \times 1k\Omega \times 1mA$$

$$= 0V.$$

3. The Rauch filter shown has a transfer function (assume ideal opamp)

$$H(s) = \frac{1}{s^2 \cdot R_1 \cdot R_2 \cdot C_1 \cdot C_2 + s \cdot C_2 \cdot \left(R_1 + R_2 + \frac{R_1 \cdot R_2}{R_3}\right) + \frac{R_1}{R_3}}.$$

If $R_1 = 2R_2 = 2R_3$ and pole Q is $Q = 1/\sqrt{3}$, what should be the ratio C_1/C_2?



For extra credit, find the requirements for the DC gain A_0 and the pole Q so that the minimum capacitance spread (C_1/C_2 or C_2/C_1) can be 1?

Solution:

$$H(s) = \frac{R_3}{R_1} \frac{\frac{1}{R_2 \cdot R_3 \cdot C_1 \cdot C_2}}{s^2 + s \cdot \frac{R_1 + R_2 + \frac{R_1 \cdot R_2}{R_3}}{R_1 \cdot R_2 \cdot C_1} + \frac{1}{R_2 \cdot R_3 \cdot C_1 \cdot C_2}}$$

The denominator for a second order transfer function is $s^2 + \frac{\omega_0}{Q}s + \omega_0^2$. Therefore,

$$\begin{cases} \frac{R_1 + R_2 + \frac{R_1 \cdot R_2}{R_3}}{R_1 \cdot R_2 \cdot C_1} = \frac{\omega_0}{Q} & (1) \\ \frac{1}{R_3 \cdot R_2 \cdot C_1 \cdot C_2} = \omega_0^2 & (2) \end{cases}$$

Since $R_1 = 2R_2 = 2R_3$, from (2) we get $\frac{1}{R_2^2 \cdot C_1 \cdot C_2} = \omega_0^2$, i.e., $\omega_0 = \frac{1}{R_2\sqrt{C_1 \cdot C_2}}$

From (1) we get $\frac{\omega_0}{Q} = \frac{2R_2 + R_2 + 2R_2}{2R_2 \cdot R_2 \cdot C_1} = \frac{5}{2R_2 \cdot C_1} = \frac{1}{R_2\sqrt{C_1 \cdot C_2}}/Q$

Thus, $\frac{C_1}{C_2} = \left(\frac{5Q}{2}\right)^2 = \frac{25}{12}$

Extra part:

The DC gain for Rauch filter is $A_0 = \frac{R_3}{R_1}$. Therefore, we have three equations.

$$\begin{cases} \dfrac{R_1 + R_2 + \dfrac{R_1 \cdot R_2}{R_3}}{R_1 \cdot R_2 \cdot C_1} = \dfrac{\omega_0}{Q} \\ \dfrac{1}{R_3 \cdot R_2 \cdot C_1 \cdot C_2} = \omega_0^2 \\ A_0 = \dfrac{R_3}{R_1} \end{cases}$$

$$\begin{cases} \left(\dfrac{A_0}{R_3} + \dfrac{1}{R_2} + \dfrac{1}{R_3}\right)\dfrac{1}{C_1} = \dfrac{\omega_0}{Q} \\ \dfrac{1}{R_3 \cdot R_2 \cdot C_1 \cdot C_2} = \omega_0^2 \end{cases}$$

$$\left(\frac{1 + A_0}{R_3} + \frac{1}{R_2}\right)^2 \frac{1}{C_1^2} Q^2 = \frac{1}{R_3 \cdot R_2 \cdot C_1 \cdot C_2}$$

$$(1 + A_0)^2 \frac{R_2}{R_3} + \frac{R_3}{R_2} + 2(1 + A_0) = \frac{1}{Q^2}\frac{C_1}{C_2}$$

In order $\frac{C_1}{C_2} = 1$ is possible, equation $(1 + A_0)^2 \frac{R_2}{R_3} + \frac{R_3}{R_2} + 2(1 + A_0) = \frac{1}{Q^2}$ must have real solution for $\frac{R_2}{R_3}$.

Let $x = \frac{R_2}{R_3} > 0$, $(1 + A_0)^2 \frac{R_2}{R_3} + \frac{R_3}{R_2} + 2(1 + A_0) = f(x) = (1 + A_0)^2 x + \frac{1}{x} + 2(1 + A_0)$.

The range of $f(x)$ for $x > 0$ is $[4(1 + A_0), +\infty)$. $f(x)$ has its minimum value when $x = \frac{1}{1+A_0}$.

Therefore, $f(x) = \frac{1}{Q^2}$ has solution when $\frac{1}{Q^2} \geq 4(1 + A_0)$.

The requirement for A_0 and Q is $Q \leq \frac{1}{2\sqrt{1+A_0}}$.